State Estimation

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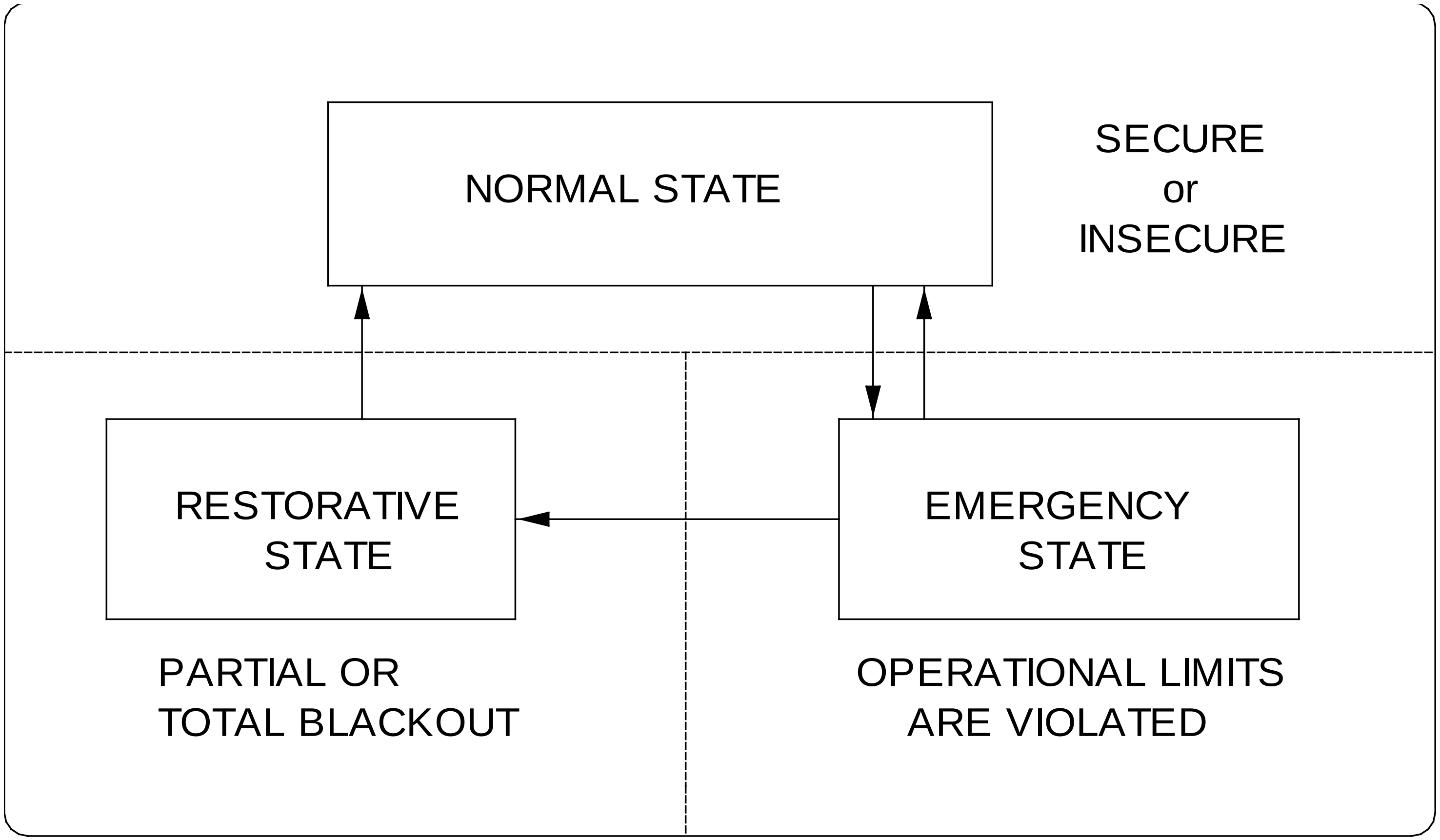
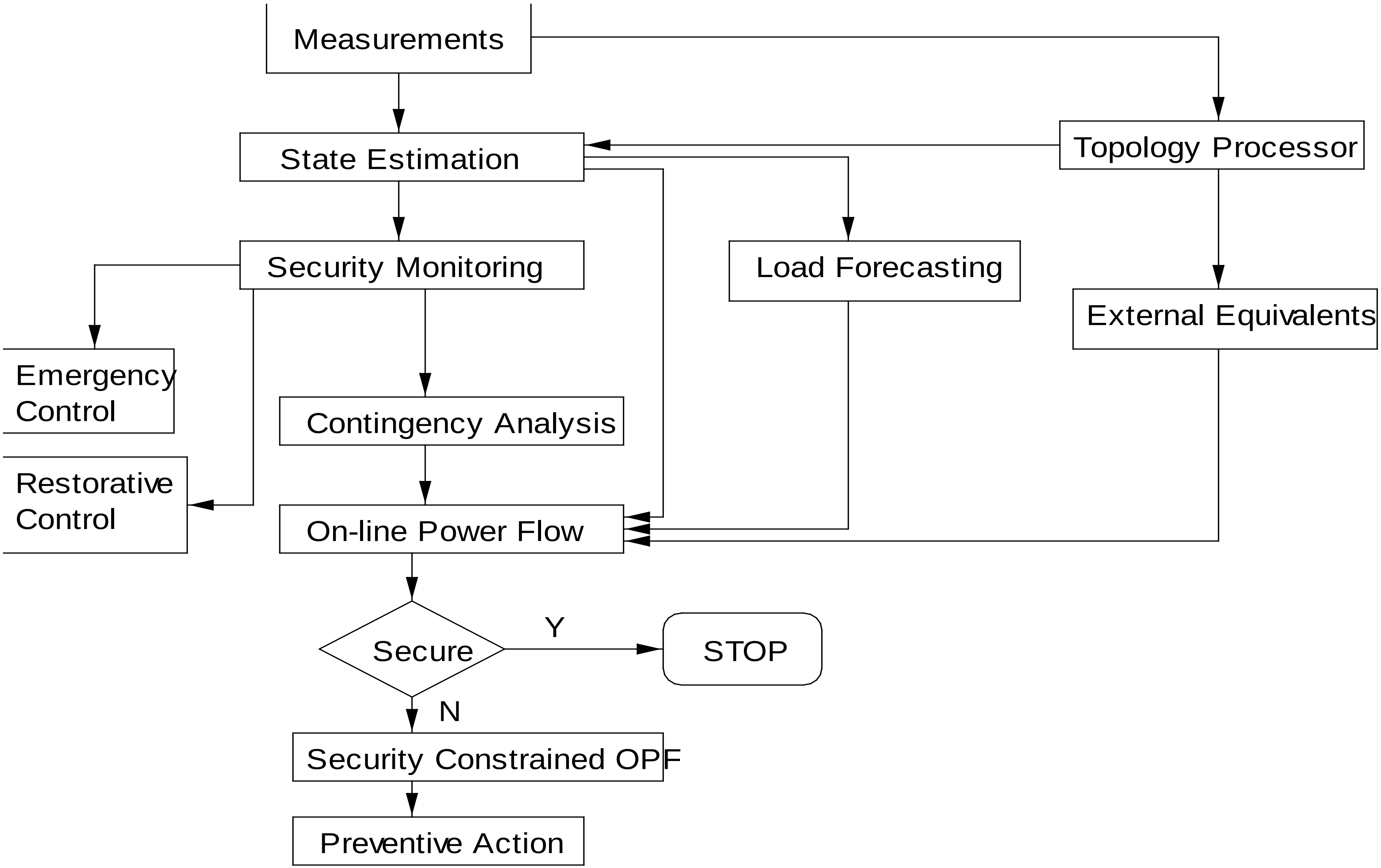
Fall 2016 CURENT Course Lecture Notes

Operating States of a Power System



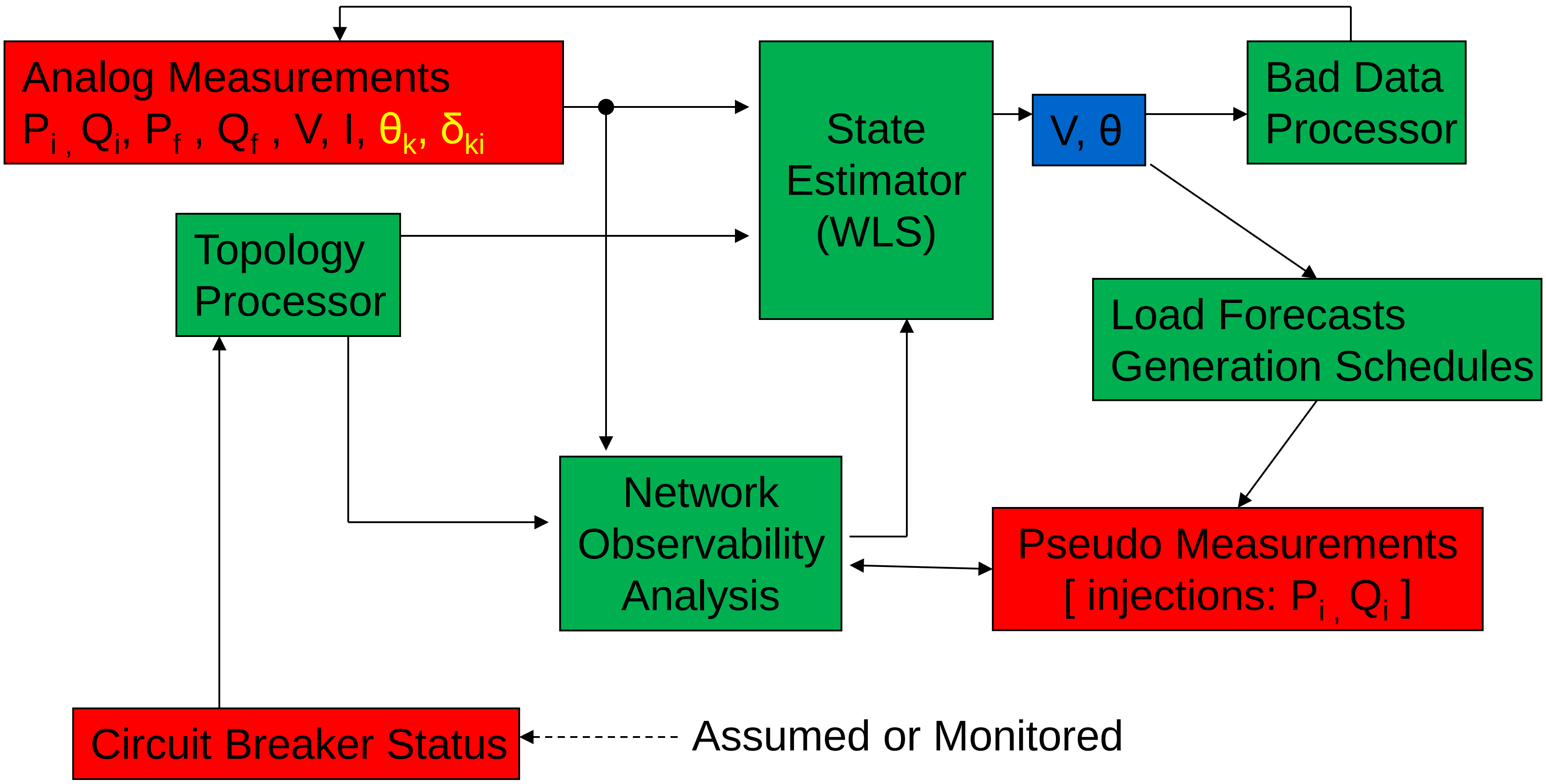
Energy Management System Applications

SCADA / EMS Configuration



State Estimation and Related Functions

Weighted Least Squares (WLS) Estimator



Power Flow and State Estimation

Comparison

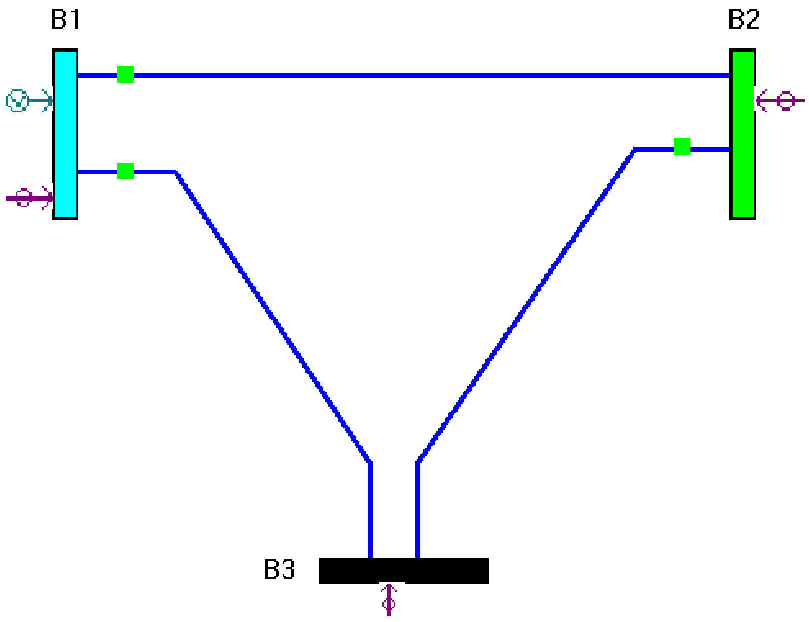
	Power Flow	State Estimation
Termination criterion	Bus P,Q mismatch	ΔX State increment
Formulation	Deterministic	Stochastic
Solution	Depends on choice of slack bus	Independent of reference bus selection
Bus Types	Important	Irrelevant
Loads	Modeled	Not used
Generator Limits	Modeled	Not used
Transmission System	Modeled	Modeled

Power System State Estimation

Problem Statement

- [z] : Measurements
  P-Q injections
  P-Q flows
  V magnitude, I magnitude

- [x] : States
  V, θ, Taps (parameters)



- **EXAMPLE:**

- [z] = [P12; P13; P23; P1; P2; P3; V1; Q12; Q13; Q23; Q1; Q2; Q3]
  m = 13 (no. of measurements)

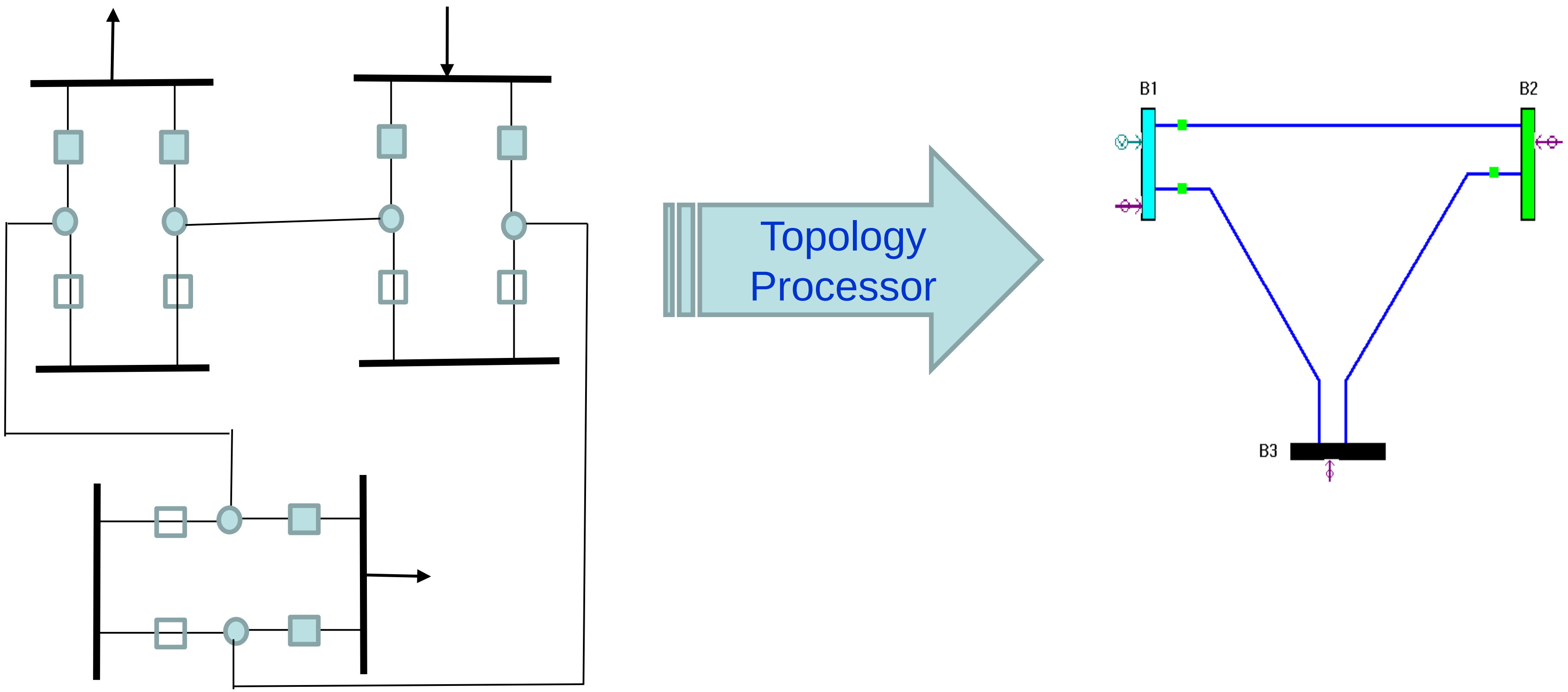
- [x] = [V1; V2; V3; θ2; θ3]
  n = 5 (no. of states)

Network Model

Bus/branch and bus/breaker Models

Bus/Breaker

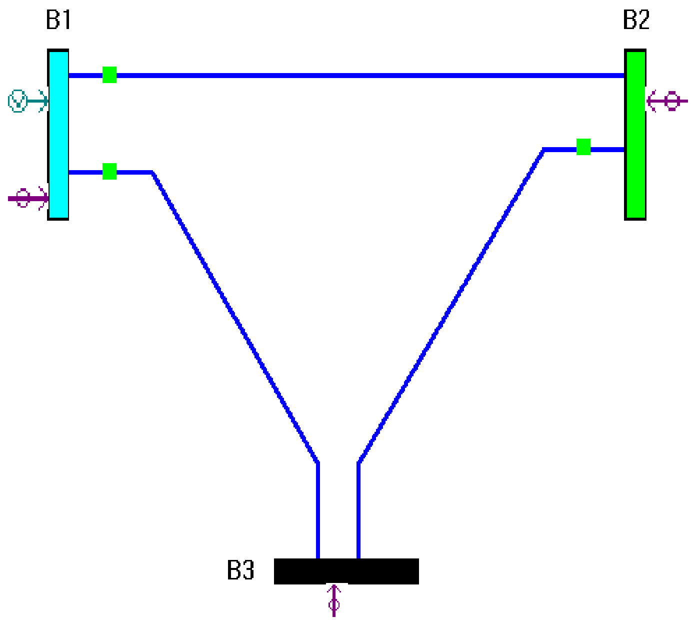
Bus/Branch



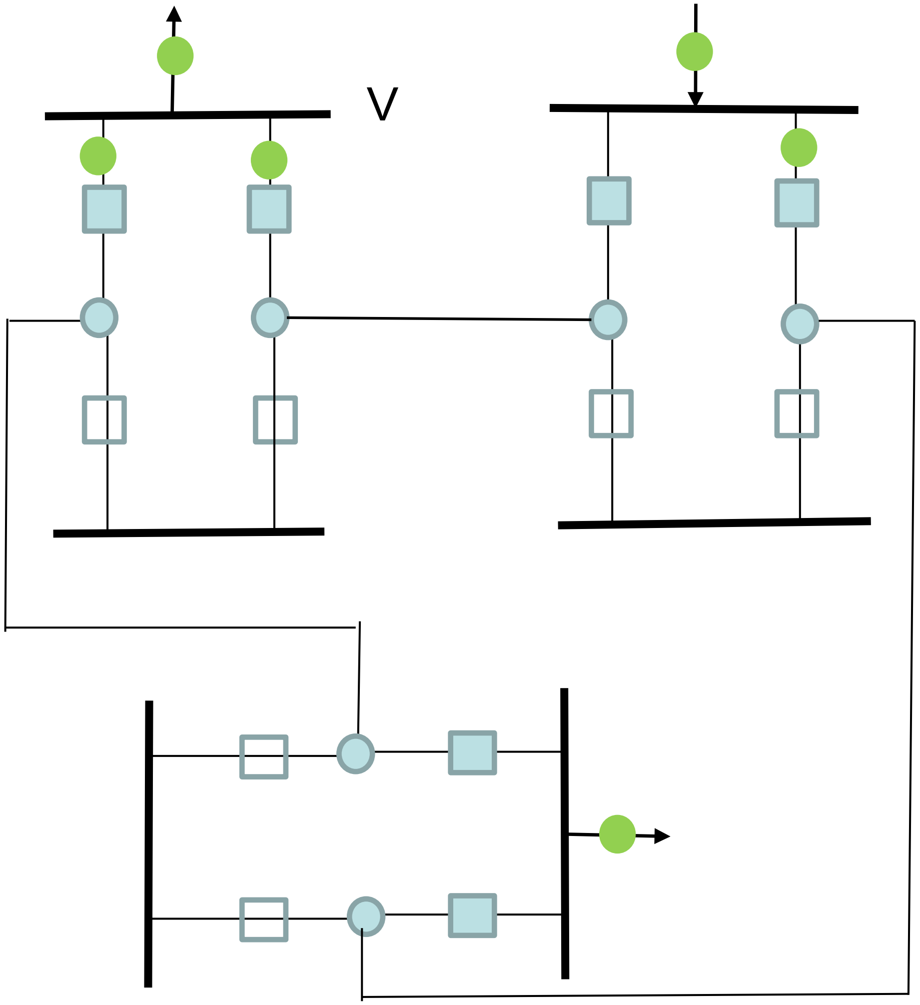
Measurements

Bus/branch and bus/breaker Models

Bus/branch

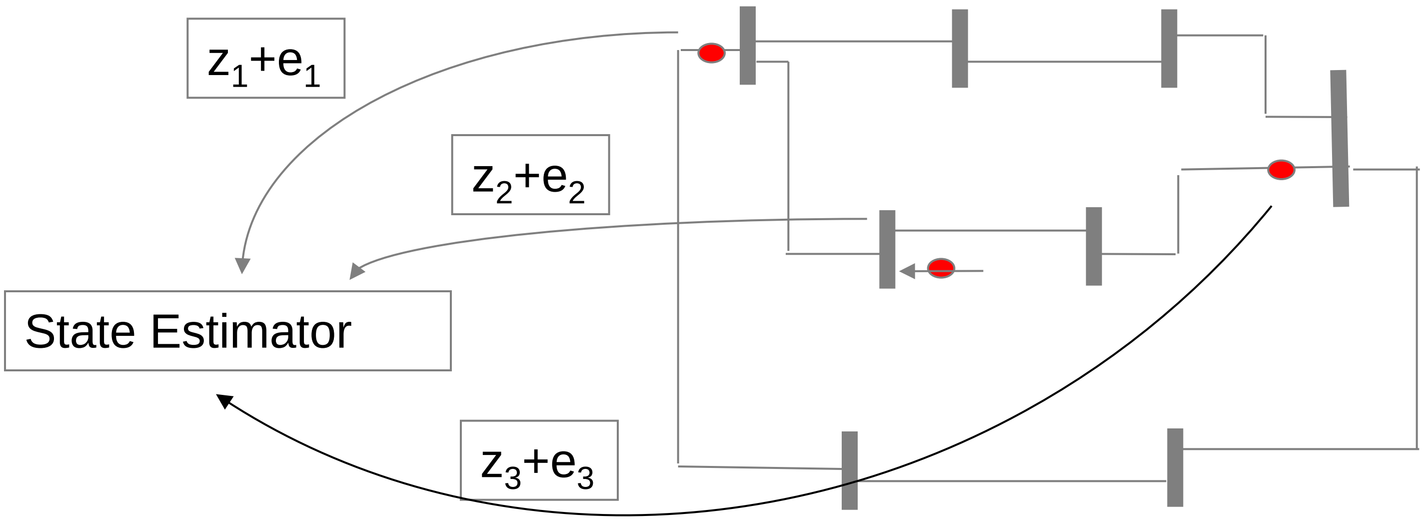


Bus/Breaker



Measurement Model

$[z_m] = [h([x])] + [e]$



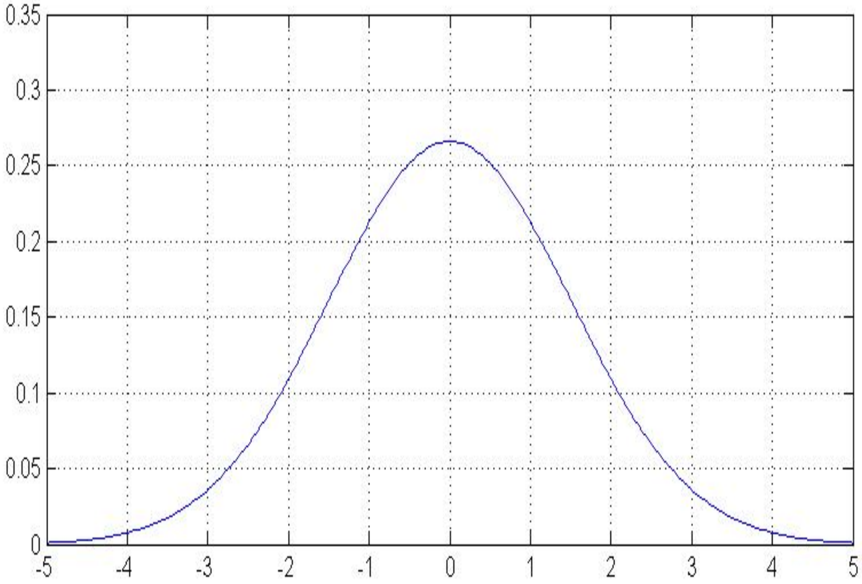
z_i : true measurement

e_i : measurement error

$e_i = e_s + e_r$

systematic random

Measurement Model



Assumptions

- $e_i \sim N\ (\ 0, \sigma_i^2\ )$
- Holds true if:

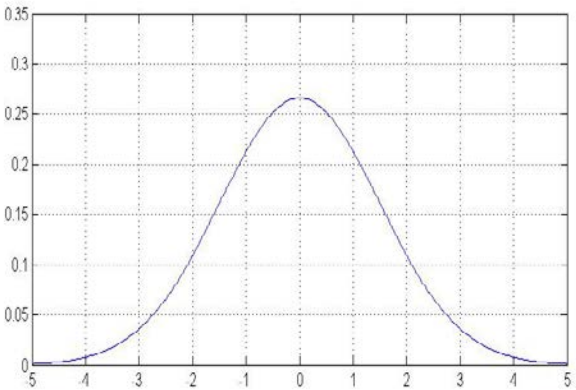
$$e_s = 0,\ e_r \sim N\ (\ 0, \sigma_i^2\ )$$

- If $e_s \neq 0$, then $E(e_i) \neq 0$,

i.e. SE will be biased !

Maximum Likelihood Estimator (MLE)

Likelihood Function



Consider the random variables $X_1, X_2, \ldots, X_n$ with a p.d.f of $f(X \mid \theta)$, where **θ** is unknown.

The joint p.d.f of a set of random observations

$$x = \{ x_1, x_2, \ldots , x_n \}$$

will be expressed as:

$$f_n(x \mid \theta) = f (x_1 \mid \theta) f(x_2 \mid \theta) \ldots f(x_n \mid \theta)$$

This joint p.d.f is referred to as the ***Likelihood Function***.

The value of **θ**, which will maximize the function **fn(x | θ)** will be called the ***Maximum Likelihood Estimator (MLE)*** of **θ**.

Maximum Likelihood Estimator (MLE)

Maximum Likelihood Estimator

Normal (Gaussian) Density Function, f(z)

$$f(z) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\tfrac{1}{2}(\tfrac{z-\mu}{\sigma})^2\}$$

Likelihood Function, $f_m(z)$

$$f_m(z) = f_m(z_1) f_m(z_2) \cdots f_m(z_m)$$

Log-Likelihood Function, *L*

$$L = \log f_m(z) = \sum_{i=1}^{m} \log f(z_i)$$

$$= -\tfrac{1}{2} \sum_{i=1}^{m} (\tfrac{z_i - \mu_i}{\sigma_i})^2 - \tfrac{m}{2} \log 2\pi - \sum_{i=1}^{m} \log \sigma_i$$

Maximum Likelihood Estimator (MLE)

Weighted Least Squares (WLS) Estimator

Given the set of observations $z_1, z_2, \ldots, z_n$ MLE will be the solution to the following:

$$\text{Maximize } f_m(z)$$

OR

$$\text{Minimize } \sum_{i=1}^{m} \left(\frac{z_i - \mu_i}{\sigma_i} \right)^2$$

Defining a new variable "r", measurement residual:

$$\text{Minimize} \sum_{i=1}^{m} W_{ii} r_i^{\,2}$$

$$\text{Subject to } z_i = h_i(x) + r_i \quad i = 1,..,m$$

$$W_{ii} = \frac{1}{\sigma_i^2}$$

$$\mu_i = E(z_i) = h_i(x)$$

The solution of the above optimization problem is called the **weighted least squares (WLS)** estimator for **x**.

Maximum Likelihood Estimator (MLE)

Weighted Least Squares (WLS) Estimator

Linear case:

$$\text{Minimize} \sum_{i=1}^{m} W_{ii} r_i^2$$

$$\text{Subject to } [z] = [H] \cdot [x] + [r]$$

Solution is given by:

$$[\hat{x}] = [G^{-1}] \cdot [H^T] \cdot [W] \cdot [z]$$

$$[G] = [H^T] \cdot [W] \cdot [H]$$

$$W_{ii} = \frac{1}{\sigma_i^2} \qquad W = diag\{W_{ii}\}$$

Measurement Model

Given a set of measurements, [z]

and the correct network topology/parameters:

$$[z] = [h([x])] + [e]$$

Measurements: Contain errors

True System States: Unknown !

Measurement Errors: Unknown !

Measurement Model

Following the state estimation, the estimated state will be denoted by $[\hat{x}]$:

$$[z] = [h([\hat{x}])] + [r]$$

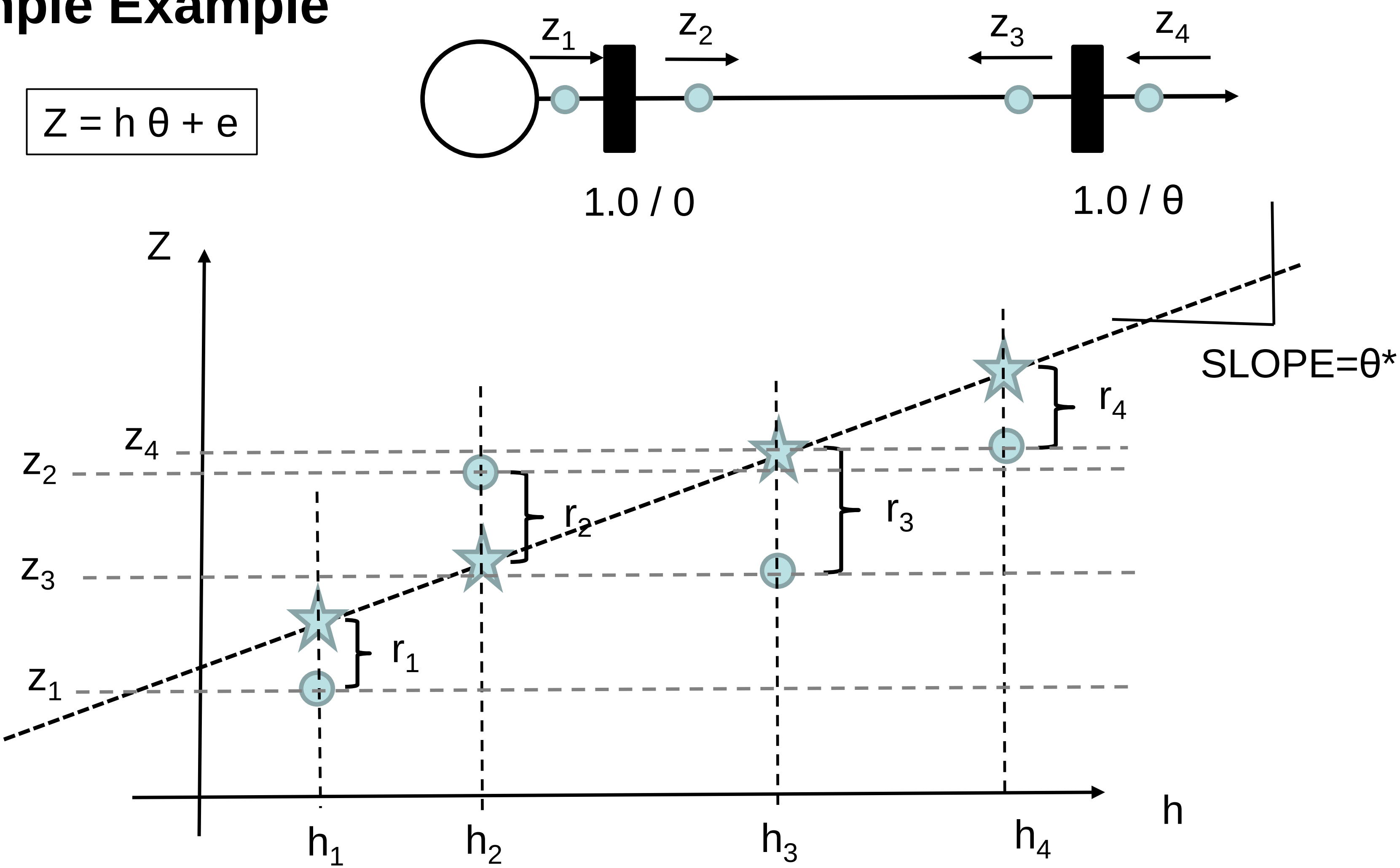
Measurements: Contain errors

Estimated System States

Measurement Residuals: Computed

Simple Example

$Z = h\,\theta + e$




: ESTIMATED MEASUREMENT


: MEASURED VALUE

r_i : MEASUREMENT RESIDUAL = $Z - h\,\theta^*$

Weighted Least Squares (WLS) Estimation

$$\textit{Minimize} \quad \omega_1 r_1^2 + \omega_2 r_2^2 + \omega_3 r_3^2 + \omega_4 r_4^2$$

What are weights, w_i ?

$$\omega_i = \frac{1.0}{\sigma_i^2}$$

How are they chosen ?

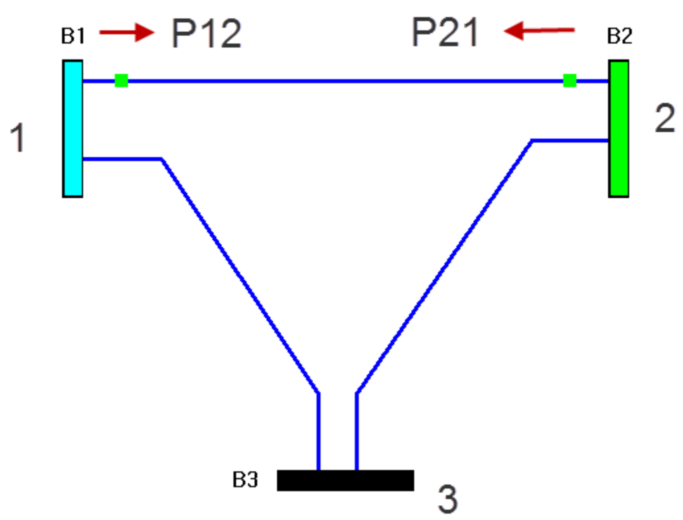
σ_i^2 Assumed error variance of measurement "*i*".

Fully observable network:

A power system is said to be ***fully observable*** if voltage phasors at all system buses can be uniquely estimated using the available measurements.

Network Observability

Necessary and Sufficient Conditions



$$Z_p = H \cdot \theta$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} H_{11} & \cdots & H_{1n} \\ H_{21} & \cdots & H_{2n} \\ H_{31} & \cdots & \vdots \\ \vdots & \cdots & \vdots \\ H_{m1} & \cdots & H_{mn} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}$$

$$\textbf{State Vector} = [\theta_2 \quad \theta_3]$$

$$\hat{\theta} = [G^{-1}] \cdot [H^T] \cdot [W] \cdot [Z_P]$$

Singular Matrix !
Cannot be inverted.

m ≥ n ➔ NECESSARY BUT “NOT” SUFFICIENT

EXAMPLE: m = 2, n = 2, UNOBSERVABLE SYSTEM

Rank(H) = n → SUFFICIENT

Measurement Classification

Types of Measurements

1. <u>CRITICAL MEASUREMENTS</u>

- *If they <u>are lost</u> or temporarily unavailable, the system will <u>no longer be observable</u>, thus state estimation can not be executed*
- *If they have gross errors, they <u>can not be detected</u>*
- *Measurement <u>residuals</u> will always be equal to <u>zero</u>, i.e. critical measurements will be perfectly satisfied by the estimated state*

2. <u>REDUNDANT MEASUREMENTS</u>

CAN BE REMOVED WITHOUT AFFECTING NETWORK OBSERVABILITY

Unobservable branch:

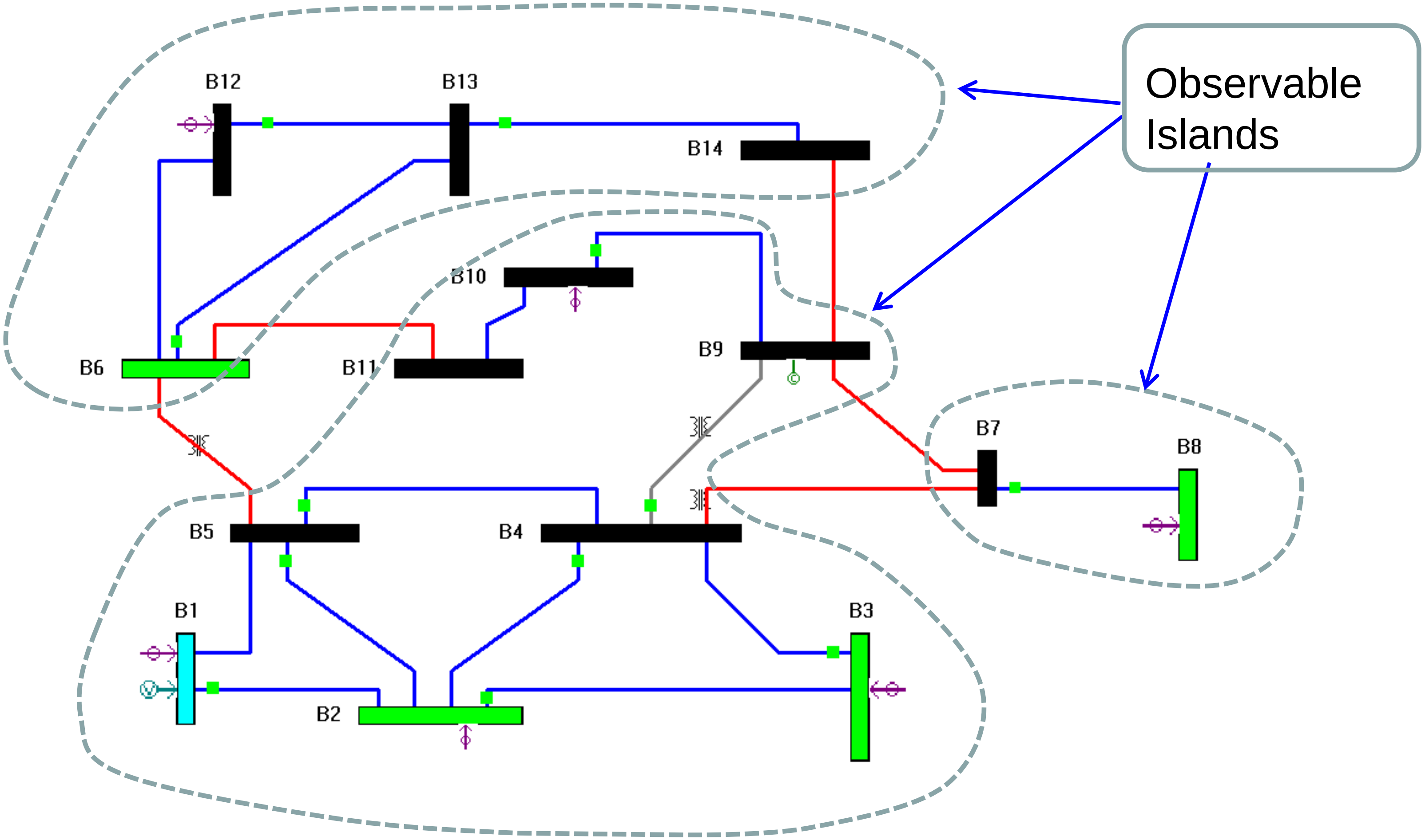
- If the system is found ***not*** to be observable, it will imply that there are ***unobservable*** branches whose power flows can not be determined.

Observable island:

- ***Unobservable*** branches connect ***observable*** islands of an ***unobservable*** system. State of each observable island can be estimated using any one of the buses in that island as the reference bus.

Network Observability

Definitions



RED LINES: Unobservable Branches

Merging Observable Islands

Pseudo-measurements

If the system is found unobservable, use pseudo-measurements in order to merge observable islands.

Pseudo-measurements:

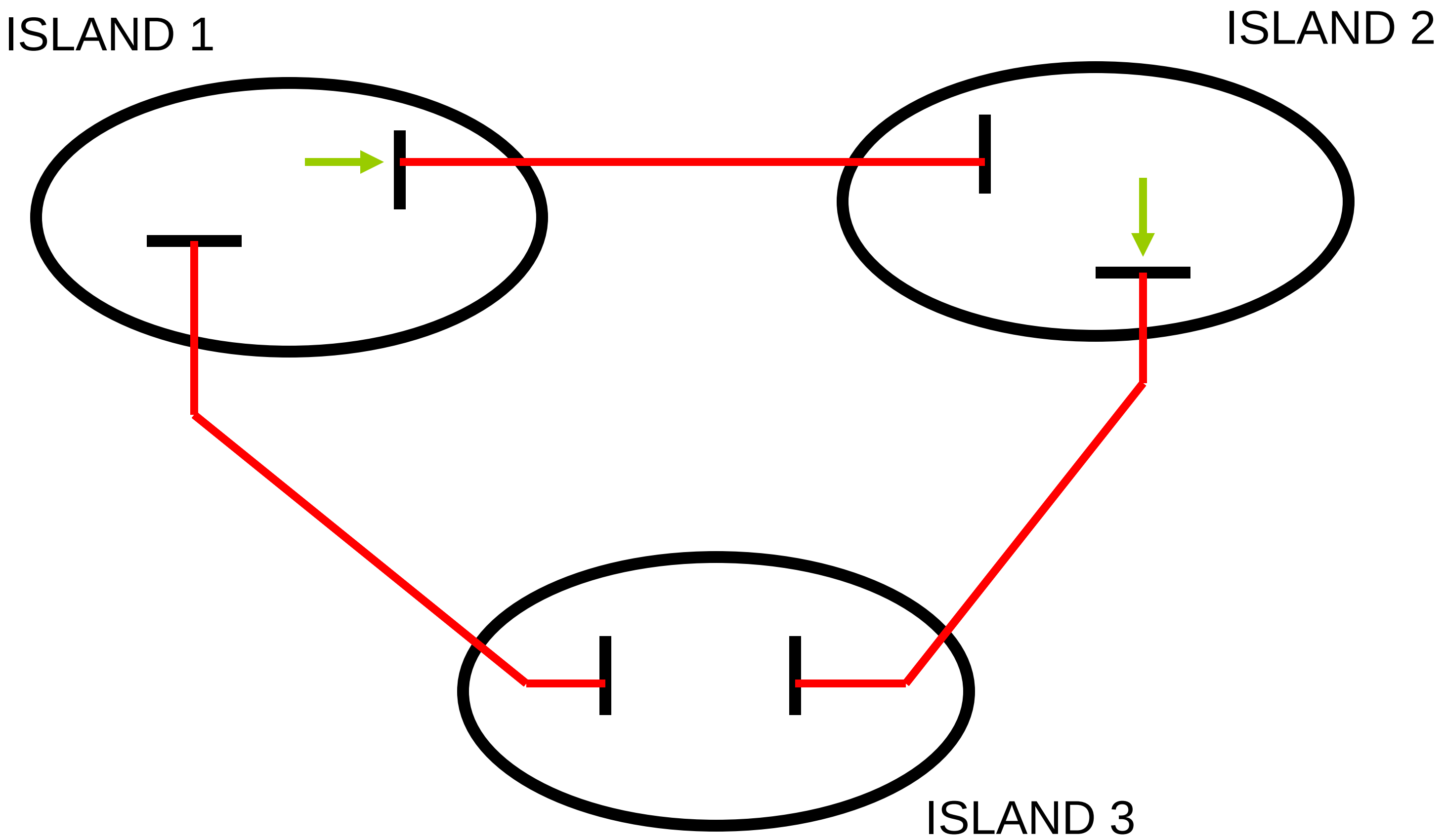
- Forecasted bus loads
- Scheduled generation

Select pseudo-measurements such that they are critical.

Errors in critical measurements do not propagate to the residuals of the other (redundant) measurements.

Observable Islands

Unobservable Branches



If an estimator remains insensitive to a finite number of errors in the measurements, then it is considered to be ***robust***.

Example: Given z = { 0.9, 0.95, 1.05, 1.07, 1.09 }, estimate z using the following estimators:

1. $\hat{X}_a = mean\{z_i\} = \frac{1}{5}\sum_{i=1}^{5} z_i$

2. $\hat{X}_b = median\{z_i\}, \quad i = 1,\ldots,5$

Solution:
Replace z_5=1.09 by an infinitely large number $z'_5 = \infty$.

The new estimate will then be: $\hat{X}'_a = \frac{1}{5}\sum_{i=1}^{5} z_i = \infty$
This estimator is NOT robust.

Replace both z_5 and z_4 by infinity.

The new estimate will then be: $\hat{X}'_b = 1.05 \ \text{(finite)}$
This is a more robust estimator than the one above.

M-Estimators (Huber 1964)

Consider the problem:

$$\text{Minimize } \sum_{i=1}^{m} \rho(r_i)$$

$$\text{Subject to } z = h(x) + r$$

Where $\rho(r_i)$ is a chosen function of the measurement residual

In the special case of the WLS state estimation:

$$\rho(r_i) = \frac{r_i^2}{\sigma_i^2}$$

Robust Estimation

M-Estimators

Some Examples of M-Estimators

Quadratic-Constant

$$\rho(r_i) = \begin{cases} \dfrac{r_i^2}{\sigma_i^2} & \left|\dfrac{r_i}{\sigma_i}\right| \le a \\ \dfrac{a_i^2}{\sigma_i^2} & \text{otherwise} \end{cases}$$

Quadratic-Linear

$$\rho(r_i) = \begin{cases} \dfrac{r_i^2}{\sigma_i^2} & \left|\dfrac{r_i}{\sigma_i}\right| \le a \\ 2a\sigma_i\,|r_i| - a^2\sigma_i^2 & \text{otherwise} \end{cases}$$

Least Absolute Value (LAV)

$$\rho(r_i) = |r_i|$$

Robust Estimation

LAV Estimator Example

Measurement Model: $$z_i = A_{i1}x_1 + A_{i2}x_2 + e_i \quad i = 1,\ldots,5$$

Measurements:

i	*Z_i*	A_{i1}	A_{i2}
1	-3.01	1.0	1.5
2	3.52	0.5	-0.5
3	-5.49	-1.5	0.25
4	4.03	0.0	-1.0
5	5.01	1.0	-0.5

LAV estimate for x
and measurement residuals:

$$\mathbf{x}^T = [3.005; -4.010]$$

$$\mathbf{r}^T = [\mathbf{0.0};\ 0.0125;\ 0.02;\ 0.02;\ \mathbf{0.0}]$$

CHANGE measurement 5 from 5.01 to 15.01 (Simulated Bad Datum):

LAV estimate for x
and measurement residuals:

$$\mathbf{x}^T = [3.02; -4.02]$$

$$\mathbf{r}^T = [\mathbf{0.0};\ \mathbf{0.0};\ 0.045;\ 0.01;\ 9.98\ ]$$

Robust Estimation

LAV Estimator Example

Measurement Model: $z_i = A_{i1}x_1 + A_{i2}x_2 + e_i \quad i = 1,\ldots,5$

Measurements:

i	Z_i	A_{i1}	A_{i2}
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CHANGE measurement 5 from 5.01 to 15.01 (Simulated Bad Datum):

LAV estimate for x
and measurement residuals:

$$x^T = [3.02; -4.02]$$

$$r^T = [\mathbf{0.0};\ \mathbf{0.0};\ 0.045;\ 0.01;\ 9.98\ ]$$

Chi-squares χ^2 Test

Consider X_1, X_2, … X_N, a set of N independent random variables where:

$$X_i \sim N(0,1)$$

Then, a new random variable Y will have a χ^2 distribution with N degrees of freedom, i.e.:

$$\sum_{i=1}^{N} X_i^2 = Y \sim \chi_N^2$$

Bad Data Detection

Now, consider the function

$$f(x) = \sum_{i=1}^{m} R_{ii}^{-1} e_i^2 = \sum_{i=1}^{m} \left(\frac{e_i^2}{R_{ii}} \right) = \sum_{i=1}^{m} \left(e_i^N \right)^2$$

and assuming:

$$e_i^N \sim r_i^N \sim N(0,1)$$

f(x) will have a χ^2 distribution with at most **(m-n)** degrees of freedom.

In a power system, since at least **n** measurements will have to satisfy the power balance equations, at most **(m-n)** of the measurement errors will be linearly independent.

Bad Data Detection

Detection Algorithm χ^2 *--Test*

Solve the WLS estimation problem and compute the objective function:

$$J(x) = \sum_{i=1}^{m} \frac{(z_i - h_i(x))^2}{\sigma_i^2}$$

Look up the value corresponding to ***p*** (e.g. 95 %) probability and ***(m-n)*** degrees of freedom, from the Chi-squares distribution table.

Let this value be $\chi^2_{(m-n),p}$ Here: $p = \Pr\{J(x) \le \chi^2_{(m-n),p}\}$

Test if

$$J(x) \ge \chi^2_{(m-n),p}$$

If yes, then bad data are detected.

Else, the measurements are not suspected to contain bad data.

Bad Data Identification

Properties of Measurement Residuals

Linear measurement model: $\Delta\hat{\boldsymbol{x}} = (\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H})^{-1}\boldsymbol{H}^T\boldsymbol{R}^{-1}\Delta\boldsymbol{z}$

$$\Delta\hat{\boldsymbol{z}} = \boldsymbol{H}\Delta\hat{\boldsymbol{x}} = \boldsymbol{K}\Delta\boldsymbol{z}, \qquad \boldsymbol{K} = \boldsymbol{H}(\boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H})^{-1}\boldsymbol{H}^T\boldsymbol{R}^{-1}$$

K *is called the hat matrix. Now, the measurement residuals can be expressed as follows:*

$$\begin{aligned}
\boldsymbol{r} &= \Delta\boldsymbol{z} - \Delta\hat{\boldsymbol{z}} \\
&= (\boldsymbol{I} - \boldsymbol{K})\Delta\boldsymbol{z} \\
&= (\boldsymbol{I} - \boldsymbol{K})(\boldsymbol{H}\Delta\boldsymbol{x} + \boldsymbol{e}) \\
&= (\boldsymbol{I} - \boldsymbol{K})\boldsymbol{e} \quad [\text{Note that KH} = \text{H}] \\
&= \boldsymbol{S}\boldsymbol{e}
\end{aligned}$$

*where **S** is called the **residual sensitivity matrix**.*

Bad Data Identification

Distribution of Measurement Residuals

The residual covariance matrix Ω can be written as:

$$E[rr^T] = \Omega = S \cdot E[e \cdot e^T] \cdot S^T$$

$$= S \cdot R \cdot S^T = S \cdot R$$

Hence, the normalized value of the residual for measurement ***i*** will be given by:

$$r_i^N = \frac{r_i}{\sqrt{\Omega_{ii}}} = \frac{r_i}{\sqrt{R_{ii} S_{ii}}}$$

- The row/column of *S* corresponding to a critical measurement will be *zero*.

- If there is a single error in the measurement set (provided that it is not a critical measurement) the largest normalized residual will correspond to that error.

Bad Data Identification

Largest Normalized Residual Test

1. Compute the normalized residuals

2. Find k such that r_k^N is the largest among all r_i^N , $i=1,\dots,m$. .

3. If r_k^N > c=3.0, then the k-th measurement will be suspected as bad data.

   Else, stop, no bad data will be suspected.

4. Eliminate the k-th measurement from the measurement set and go to step 1.

Use of Synchrophasor Measurements

- Given enough phasor measurements, state estimation problem will become LINEAR, thus can be solved directly without iterations

$$\textit{Conventional Measurements}$$

$$Z = h(X) + e$$

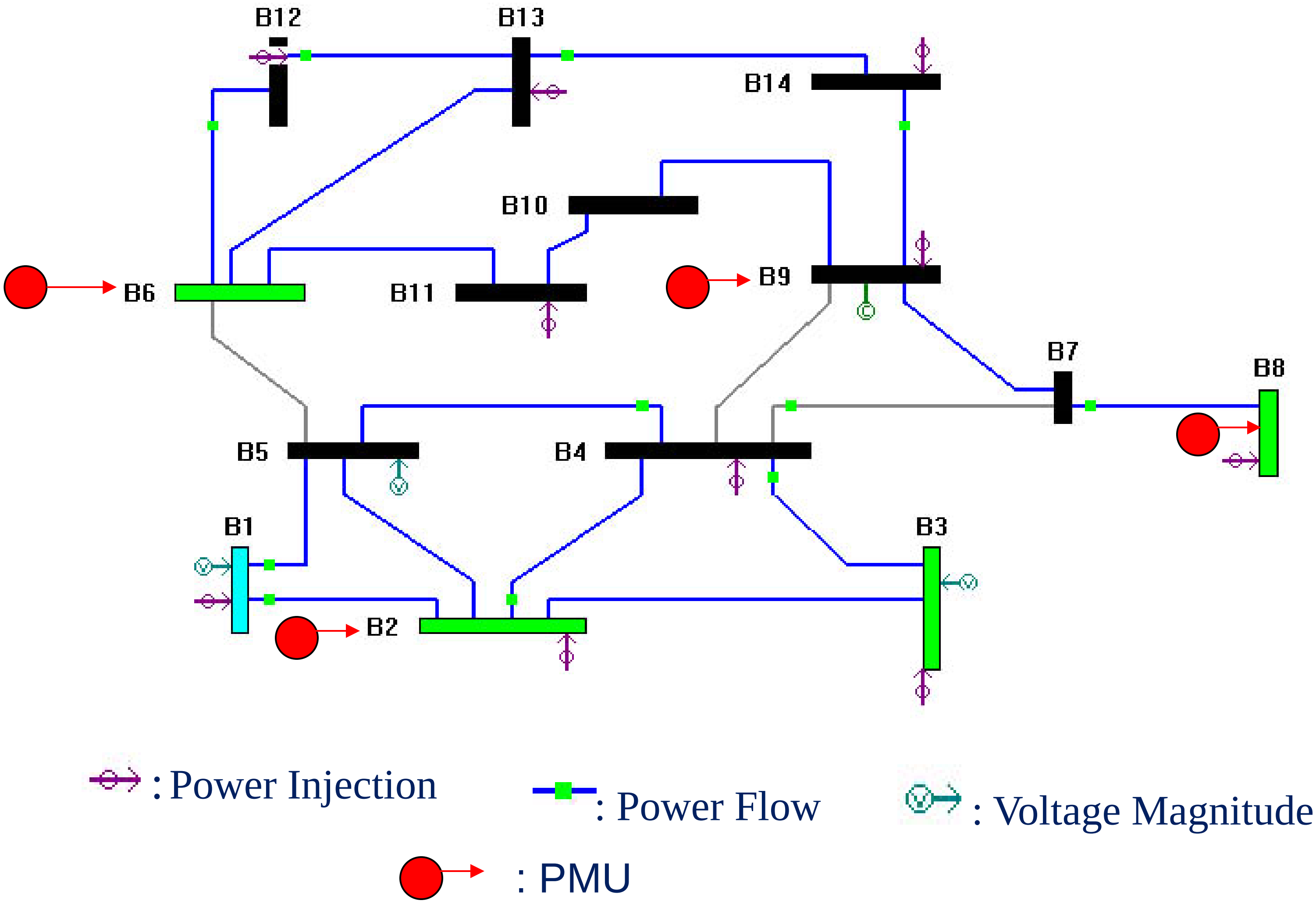
$$\Delta\hat{X} = (H^T R^{-1} H)^{-1} R^{-1} \Delta Z \quad \textit{Iterative}$$

$$\textit{Phasor Measurements}$$

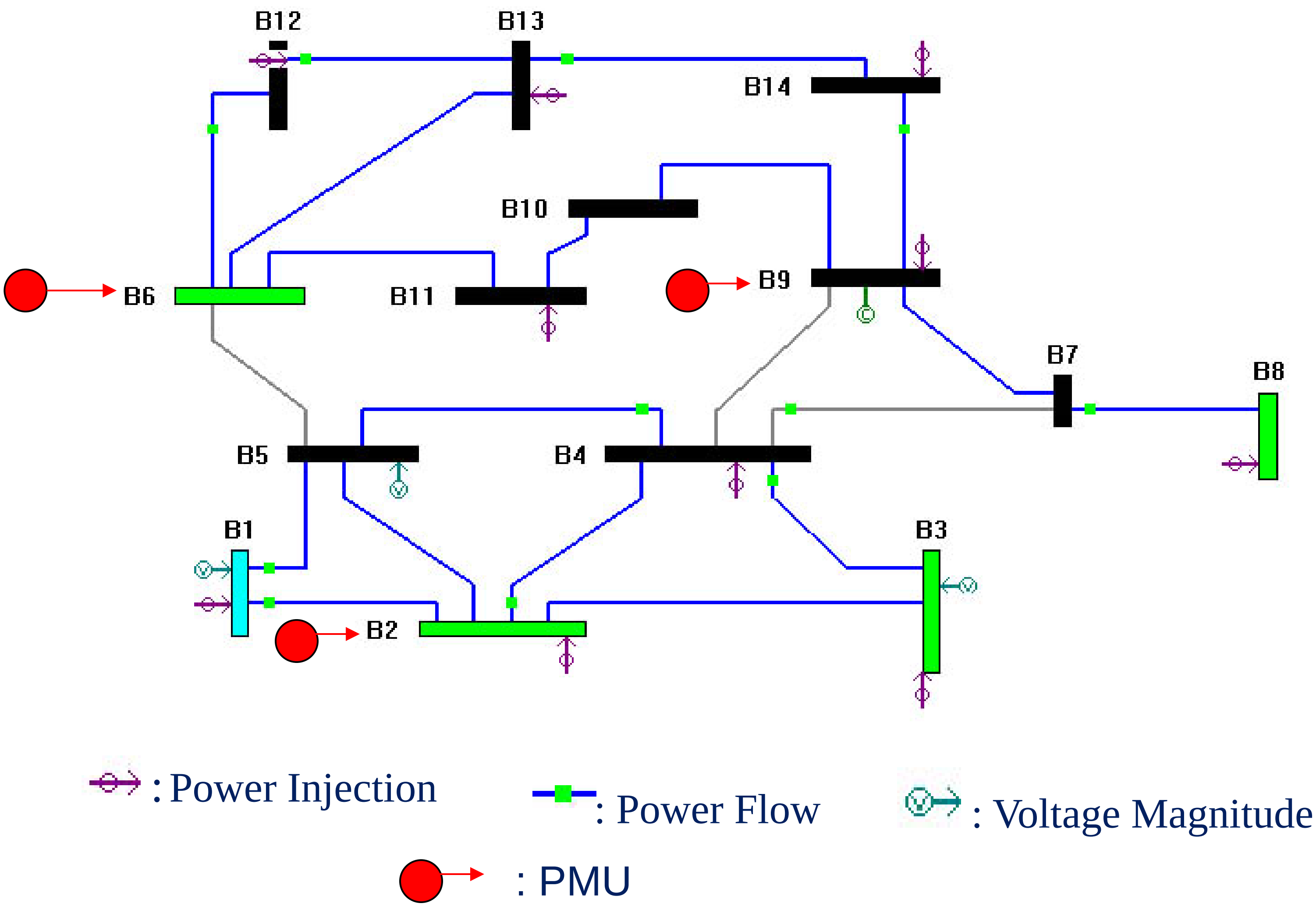
$$Z = H \cdot X + e$$

$$\hat{X} = (H^T R^{-1} H)^{-1} R^{-1} Z \quad \textit{Non-iterative}$$

Placing PMUs:



Exploiting zero injections



Use of Synchrophasor Measurements

- Given at least one phasor measurement, there will be no need to use a reference bus in the problem formulation
- Given unlimited number of available channels per PMU, it is sufficient to place PMUs at roughly 1/3rd of the system buses to make the entire system observable just by PMUs.

Systems	No. of zero injections	Number of PMUs	
		Ignoring zero Injections	Using zero injections
14-bus	1	4	3
57-bus	15	17	12
118-bus	10	32	29

Performance Metrics

- State Estimation Solution

  - <u>Accuracy:</u>

    Variance of State = inverse of the gain matrix, $[G]^{-1}$
    $= E[(x – x^*) (x – x^*)']$

  - <u>Convergence:</u>

    Condition Number = Ratio of the largest to smallest eigenvalue

    Large condition number implies an ill-conditioned problem.

Performance Metrics

- Measurement Design

  - <u>Critical Measurements:</u>

    Number of critical measurements and their types

  - <u>Local Redundancy</u>

    Number of measurements incident to a given bus

  - <u>(N-1) Robustness</u>

    Capability of the measurement configuration to render a fully observable system during single measurement and branch losses

Summary

- State Estimation and its related functions are reviewed.
- Importance of measurement design is illustrated.
- Commonly used methods of identifying and eliminating bad data are described.
- Impact of incorporating phasor measurements on state estimation is briefly reviewed.
- Metrics for state estimation solution and measurement design are suggested.

Power Education Toolbox (P.E.T)

Power Flow and State Estimation Functions

Free software to:
Build one-line diagrams of power networks
Run power flow studies
Run state estimation

http://www.ece.neu.edu/~abur/pet.html

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